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Colegiul Naţional „Constantin Diaconovici Loga" din Timişoara

Co-funded by the Erasmus+ Programme of the European Union

Project title: Computer supported Math teaching, COMPMATH
Ref. No.: KA210-SCH-ABDA6A90
Action Type: KA210-SCH - Small-scale partnerships in school education

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Maths in Architecture

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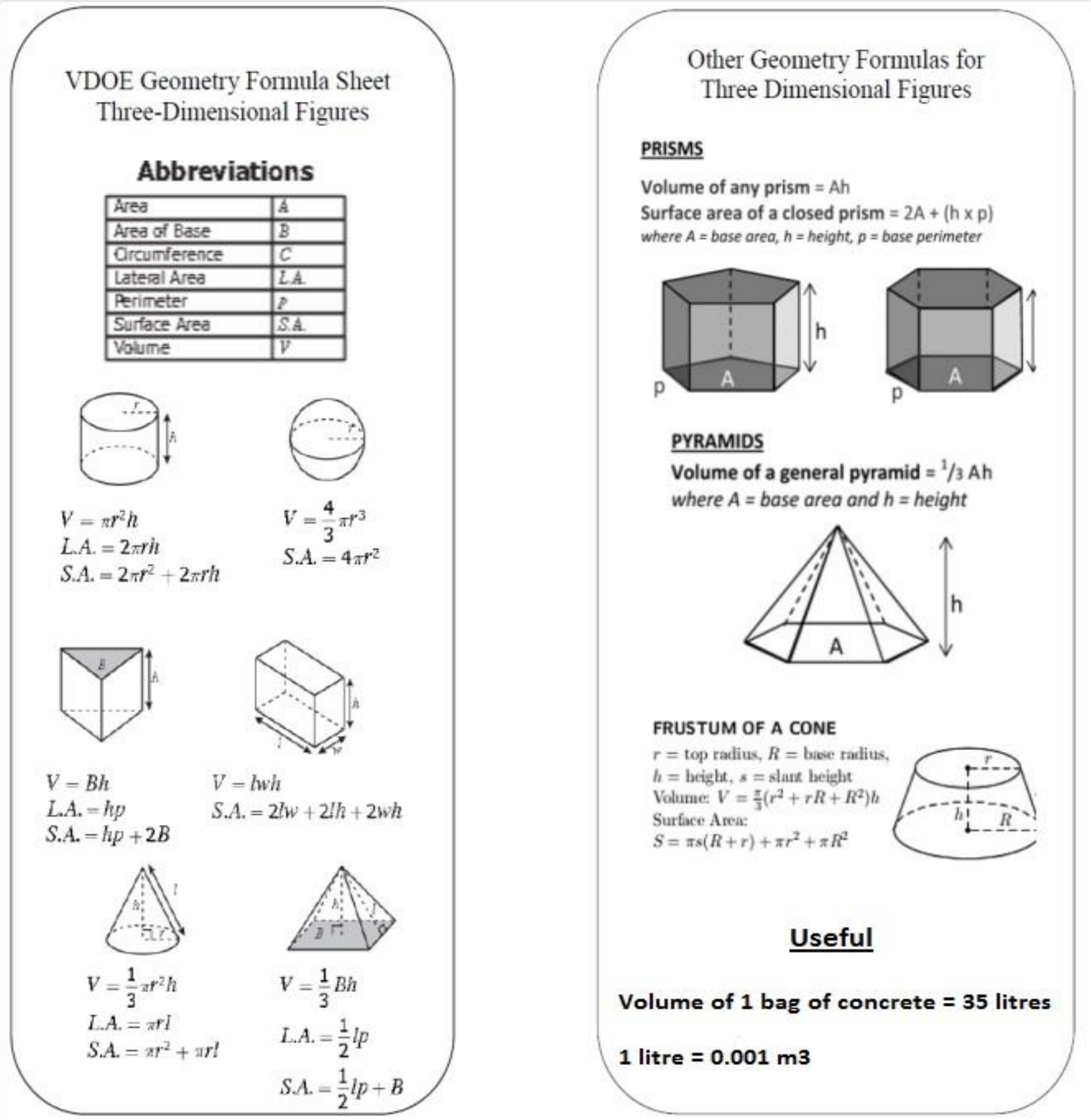
Theoretical background

Geometry Figure Formula Sheet



VDOE Geometry Formula Sheet
Three-Dimensional Figures

Abbreviations

Area	A
Area of Base	B
Circumference	C
Lateral Area	L.A.
Perimeter	p
Surface Area	S.A.
Volume	V

$V = \pi r^2 h$
$L.A. = 2\pi r h$
$S.A. = 2\pi r^2 + 2\pi r h$

$V = \frac{4}{3}\pi r^3$
$S.A. = 4\pi r^2$

$V = Bh$
$L.A. = hp$
$S.A. = hp + 2B$

$V = lwh$
$S.A. = 2lw + 2lh + 2wh$

$V = \frac{1}{3}\pi r^2 h$
$L.A. = \pi r l$
$S.A. = \pi r^2 + \pi r l$

$V = \frac{1}{3}Bh$
$L.A. = \frac{1}{2}lp$
$S.A. = \frac{1}{2}lp + B$

Other Geometry Formulas for
Three Dimensional Figures

PRISMS

Volume of any prism = Ah
Surface area of a closed prism = 2A + (h x p)
where A = base area, h = height, p = base perimeter

PYRAMIDS

Volume of a general pyramid = $^1/_3$ Ah
where A = base area and h = height

FRUSTUM OF A CONE

r = top radius, R = base radius,
h = height, s = slant height
Volume: $V = \frac{\pi}{3}(r^2 + rR + R^2)h$
Surface Area:
$S = \pi s(R + r) + \pi r^2 + \pi R^2$

Useful

Volume of 1 bag of concrete = 35 litres

1 litre = 0.001 m3

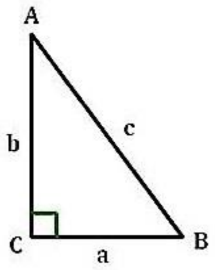
Resources:
http://www.slideshare.net/PDF-eBooks-For-Free/geometry-formulas-2d-and-3d-ebook
http://www.math-salamanders.com/image-files/high-school-geometry-help-geometry-cheat-sheet-5-3d-shape-formulas.gif
http://www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_formula_sheet_geometry.pdf
lowes.com
metric-conversions.org

Elementary Trigonometry

In a right triangle, the side opposite the right angle is called the hypotenuse, and the other two sides are called its legs. By knowing the lengths of two sides of a right triangle, the length of the third side can be determined by using the Pythagorean Theorem:

Pythagorean Theorem: The Square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of its legs.

Trigonometric Functions of an Acute Angle:



SOH-CAH-TOA

$\sin = \frac{opposite}{hypoteneuse}$ $\quad \sin A = \frac{a}{c}$ $\quad \sin B = \frac{b}{c}$

$\cos = \frac{adjacent}{hypoteneuse}$ $\quad \cos A = \frac{b}{c}$ $\quad \cos B = \frac{a}{c}$

$\tan = \frac{opposite}{adjacent}$ $\quad \tan A = \frac{a}{b}$ $\quad \tan B = \frac{b}{a}$

Local maximums and minimums

A real-valued function f defined on a domain X has a global (or absolute) maximum point at x^* if $f(x^*) \geq f(x)$ for all x in X. Similarly, the function has a global (or absolute) minimum point at x^* if $f(x^*) \leq f(x)$ for all x in X. The value of the function at a maximum point is called the maximum value of the function and the value of the function at a minimum point is called the minimum value of the function.

If he domain X is a metric space then f is said to have a local (or relative) maximum point at the point x^* if there exists some $\varepsilon > 0$ such that $f(x^*) \geq f(x)$ for all x in X within distance ε of x^*. Similarly, the function has a local minimum point at x^* if $f(x^*) \leq f(x)$ for all x in X within distance ε of x^*. A similar definition can be used when X is a topological space, since the definition just given can be rephrased in terms of neighbourhoods.

In both the global and local cases, the concept of a strict extremum can be defined. For example, x^* is a strict global maximum point if, for all x in X with $x \neq x^*$, we have $f(x^*) > f(x)$, and x^* is a strict local maximum point if there exists some $\varepsilon > 0$ such that, for all x in X within distance ε of x^* with $x \neq x^*$, we have $f(x^*) > f(x)$. Note that a point is a strict global maximum point if and only if it is the unique global maximum point, and similarly for minimum points.

A continuous real-valued function with a compact domain always has a maximum point and a minimum point. Finding global maxima and minima is the goal of mathematical optimization. If a function is continuous on a closed interval, then by the extreme value theorem global maxima and minima exist. Furthermore, a global maximum (or minimum) either must be a local maximum (or minimum) in the interior of the domain, or must lie on the boundary of the domain. So a method of finding a global maximum (or minimum) is to look at all the local maxima (or minima) in the interior, and also look at the maxima (or minima) of the points on the boundary, and take the largest (or smallest) one.

Local extrema of differentiable functions can be found by Fermat's theorem, which states that they must occur at critical points. One can distinguish whether a critical point is a local maximum or local minimum by using the first derivative test, second derivative test, or higher-order derivative test, given sufficient differentiability.

For any function that is defined piecewise, one finds a maximum (or minimum) by finding the maximum (or minimum) of each piece separately, and then seeing which one is largest (or smallest).(*Wikipedia*)

The first step to calculating the relative extrema is calculating the derivative of a function dy/dx.

After calculating the derivative (dy/dx), set the derivative equal to 0. Local extrema all have a slope of zero. Setting the derivative, which is the slope at a specific point, equal to zero shows all points where the graph has a slope of zero, and as a result is an extrema.

Solve for x.

After solving for x, you must do the First Derivative Test

A) Draw a number line with the x value(s)

B) Use numbers before and after the calculated x value, plugged into the first derivative equation,to determine whether the numbers before and after are positive or negative.

C) If the number before is positive and the number after is negative, the extrema is a local maximum.

D) If the number before is negative and the number after is positive, the extrema is a local minimum

E) On the occasion the number before and after is the same sign (positive or negative) the number is not a local maximum or minimum.

F) The reason the first derivative test is true is because the first derivative is a slope the function. The point where the slope changes from positive to negative is a maximum because the function increases as much as it possibly can before its slope equals zero (critical point) and then decreases after this point because the slope is then negative. The opposite is true for a minimum.

1. Surface area and Volume of figures

Field of application: Geometric figures.
Required knowledge: Calculations with real numbers, Percentages, Average, Comparison.
Project: The famous architectural office of Santiago Calatrava; wants to test your math skills with a series of geometric task.
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=5&sesskey=teD0rQZDlw#section-4
Authors:
Coordinator:
The assignment of this lesson requires from students to find the surface area and volume of figures, to approximate the amount and cost of materials you'll need to construct a castle, and to make predictions about changes to dimensions
Resources: Three-Dimensional Design (Model created using Google Sketchup), Two-Dimensional Views, Geometry Figure Formula Sheet .
Problem:
You and your friends are excited about working next summer in the famous architectural office of Santiago Calatrava; however, Santiago wants to test your math skills with the following geometric task.
You are given below a three-dimensional (3D) blueprint of a triangular castle design (*Figure 1*) with different geometric figures that touch (together with two-dimensional views of castle and a scale – *Figure 2*). A formula sheet and patterns to aid you in tasks will be provided to you (*Figure 3*).
The task requires from you to find the surface area and volume of figures, to approximate the amount and cost of materials you'll need to construct the castle, and to make predictions about changes to dimensions.

Three-Dimensional Design
(Model created using Google Sketchup)

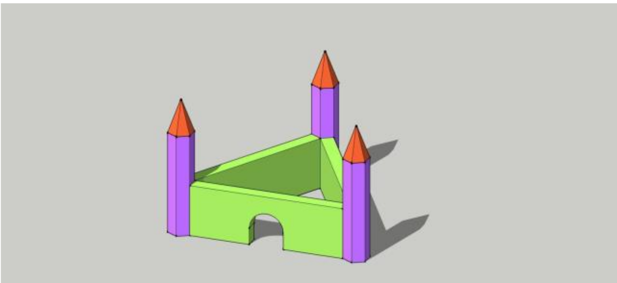


Figure 1

Two-Dimensional Views

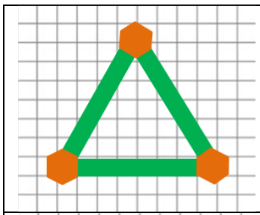
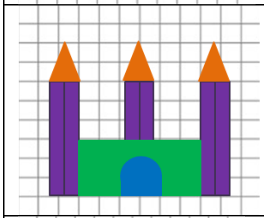
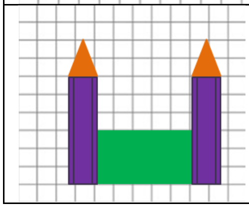
	**Top View** Legend and Scale Grid is 2cm x 2cm for model construction, 1m x 1m for real construction the scale from model to actual size is 1 to 50
	**Front View** The blue picture in the front view is the cutaway of the front rectangular prism (entrance)
	**Side View** The three green rectangular prisms have the same dimensions.

Figure 2

Task:

A) Geometric Figures Details: Discover the geometric shapes used for the construction of triangular castle. Include answers in the table below. All work including formulas used should be shown for full credit. (Hint: Google.com and mathworld.wolfram.com may help you)

Geometric Name of Figure	Surface Area equation	Volume equation

B) Analysis Questions: Include answers in the table provided below. All work including formulas used and units should be shown for full credit.

1. Find the total volume and surface area of model and actual castle. (Note: there are no bases, i.e. bottom of castle need not be constructed)

2. If a 35 liters bag of "cement" sells for €3.97, how much cement will you need to build the castle and what will it cost for cement supply? (Hint: answers should be in m^3 and euro; see Geometry Figure Formula sheet for conversion details).

3. Which shape on your castle makes the most sense to double in size without having to change the other shapes? How does the volume and surface area of this shape change when the dimensions are doubled? Express your answer as a ratio comparing the old and new results (although the total surface area and volume of castle may change, this question only pertains to one figure).

Geometric Name of Figure	Dimensions of Figure (include units)		Surface Area (include units)		Volume (include units)	
1.						
2.						

2. Stadium seating

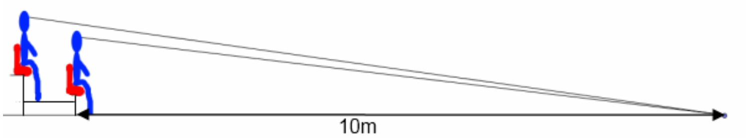
Field of application: Geometry, model a real-life architecture situation mathematically
Required knowledge: slope calculation, spreadsheet, arithmetic and geometric Sequences and Series
Project: Design the seating layout for a stadium.
Moodle:http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=5#section-2
Authors:
Coordinator:
The assignment: This problem encourages students to model a real-life architecture situation mathematically, that of tiered seating design in sports stadium.

Background:In stadium seating, most or all seats are placed higher than the seats immediately in front of them so that the occupants of further-back seats have less of their views blocked by those further forward. This is especially necessary in stadiums where the subject matter is typically best observed from above, rather than in-line or from below. In addition to sports venues and performing arts venues, many other venues that require clear audience views of a single area use stadium seating, including religious institutions, lecture halls, and movie theaters

1st Assignment - Assumptions and height of 2nd seat
One of the challenges in designing a stadium is to make sure that spectators can see the event without their views being blocked by the spectators in front.

Your task is to design the tiered seating for the stadium.

The stadium director has stipulated that all spectators must be able to see clearly a point 10m in front of the front row of seating:



The spectator in the second row needs to have line of sight to the same point as the spectator in the first row, as seen in the diagram above. Notice that the spectator in the second row needs some extra clearance in order to see comfortably over the first spectator's head.

Your tasks

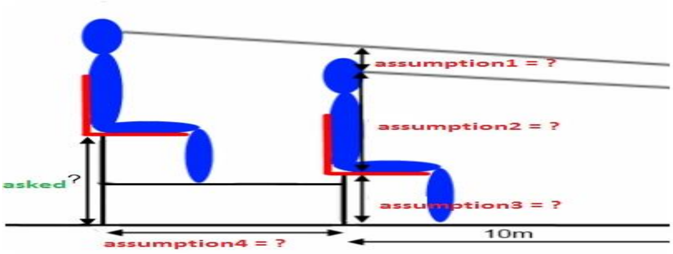
1) Go to edpuzzle.com and create a student account if haven't yet (no email address needed!). Click on "Join Class" and enter the DREAM class code: atpakde. Watch the video about math modeling, answering where necessary.

2) Make reasonable assumptions about the following items:

- The first spectator's eye-level is .... m above the ground.
- There is an extra ..... m of clearance from his eye-level to the second spectator's line-of-sight, so that each row can see over the row in front.
- The back of each seat is ..... cm behind the back of the seat in front.

And sketch the diagram that shows your assumptions.

3) How high above ground level does spectator 2's seat need to be?



2nd Assignment
Now draw a similar diagram with the dimensions and unknowns for spectators 2 and 3.
How high above ground level does spectator 3's seat need to be?

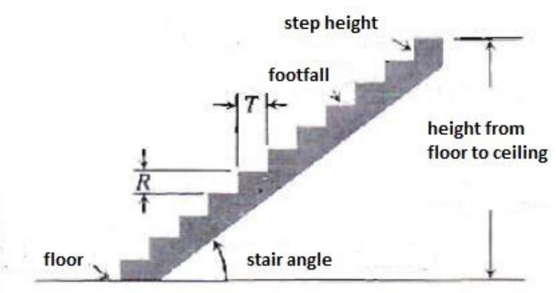
3rd Assignment
Finally, imagine there were 40 rows of seating in the stadium.
Can you work out the heights above ground level of each of the 40 rows, and hence plot a side view of the seating?
It is very helpful to use a spreadsheet to perform the repeated calculations and plot the results.

Resources: https://en.wikipedia.org/wiki/Stadium_seating
https://edpuzzle.com/

Generalization: design a seating plan for the new football arena with a number of seats between 18000 and 22500 .

3. The house stair slope

Field of application: Geometry
Required knowledge: angles, fraction, trigonometric functions
Project: The house stair slope
Moodle:http://srv-1lyk-aigiou.ach.sch.gr/moodle/mod/assign/view.php?id=163
Authors:
Coordinator:
The problem: The figure shows the intersection of a house stair, where R and T express the height and the step of the stair respectively



- Find the minimum and maximum angles of a house stair
- A typical angle for house stairs is 40o. If the step of the foot is 23cm, find the stair height
- According to architects, the most "suitable" ratio R/T is 16cm / 30cm. Find the tilt of the "ideal" stair

Resources: Moodle
Generalization: Research can be extended to other stairs.

4. Bypass for a small village

Field of application: Geometry, physics
Required knowledge: Measurement, ratio and proportion and speed-time-distance calculations.
Project: Bypass for a small village
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=5&sesskey=od6JiQPi0G#section-4
Authors:
Coordinator:
The assignment:.
This case study is designed for students to discover how useful and necessary mathematical ideas can be when solving real-world architecture problems. In particular, this case study challenges students to create an optimal route for a village bypass using software, while ensuring that the proposed route conforms to the mathematical conditions laid down by the Highways Agency and also considering social, environmental and cost minimization issues. Mathematics can provide justification for positioning a new a village bypass highway. The pupils are also challenged to make decisions about priorities and to discuss the strengths and weaknesses of others' routes.
Measurement, ratio and proportion and speed-time-distance calculations are needed to design the bypass. The emphasis is on using mathematical findings and reasoning to convince others. There are no 'right' answers; routes are presented to the class and put to a peer assessment.

*In particular, students will first consider all the issues that need to be taken into account in a bypass (**Lesson 1**) and then propose, measure and refine routes for the bypass (**Lesson 2**). Then (**Lesson 3**) groups will share their proposals and a class peer assessment will decide for the better group. (**Lesson 4)** will be devoted to ask the students to reflect on the case study and tell them to design bypass for a local town or village that arguably needs one and analyse it using Google Maps or Google Earth.*

Students consider the social issues of bypasses, their benefits and drawbacks, through relative newspaper articles.Arised issues of financial costs, safety, health and pollution, traffic flow and impacts on the natural and human environment are considered. Controversial bypasses have resulted in public campaigns either for or against a given bypass.
Students are expected to understand that length, number of junctions and terrain affect the cost of a bypass; and that curves and junctions slow traffic down.
The bypass must be safe and allow traffic to flow freely and costs must be kept to a minimum. There is no right answer and the best solution will be decided by class.We distribute Curves and Speed Limits, Costs and Junctions papers to help students designing the optimal bypass. Each group will review and refine their rough route from last lesson using these resources.
The problem: Does Halstead Need a Bypass?
"Halstead is a small market town in South East England, North Essex, and within 15 miles west of Colchester. Halstead is next to the River Colne, and is situated in the Colne Valley. Halstead has a population of 10 000 and is also the only settlement of its size in the Essex region without a bypass. Halstead was also a weaving town (where sheep's wool is made into clothe).
Halstead is central to several big towns, such as Colchester, Braintree and Haverhill. Everyday traffic from all these towns has to pass through Halstead high street in order to commute, this usually results in Halstead becoming greatly congested on a regular basis, increasing air and noise pollution, and therefore Halstead central could hugely benefit from a bypass.
A bypass is a route, which is built to avoid or 'bypass' congestion in a built up town or village, this lets traffic flow without interferences from local traffic, this improves congestion and road safety.
There are many reasons for and against the construction of a bypass.
Advantages:

- Less congestion in town.
- Less pollution in town central.
- Lorries would no longer have to drive through the town.
- Both noise and air pollution would decrease in town.
- Local builders would hugely benefit, from work needed.
- It will be quicker for people to travelling to work.

Disadvantages:

- Expensive, local tax payers of Halstead would compensate.
- An increase in noise pollution.

An increase in air pollution.
Bypass would destroy surrounding environment.
Less customers and income from commuters in the town centre.

In the past there have been several proposals for a bypass to be built in Halstead. In 1987, the Essex County Council classified main roads into the town a 'busy' and found that nearly 50 % of the traffic was passing through Halstead. A bypass was first proposed in 1990, and public discussions were held, and a preferred route emerged, although it was the most costly of all options, costing £11 million. Four Years later two small changes were made to the route, following further discussions. Later, in 1997, it was decided that a bypass may be built after 2000, when sufficient funds may become available, but has continued to be put on hold.

After analysing my results I conclude that I do not believe that a bypass should be built in Halstead. I think that he environmental impact, and impact on local residents is too big. Taxes would increase enormously and the consequence on businesses in the town will also be huge. The environment would be destroyed, ruining many habitats and bridle paths so horse riders and hikers wouldn't benefit at all.

Although I don't believe Halstead needs a bypass, if one were to be built, I reckon that route A is the best choice, as this route goes further around the town than route B, therefore the town of Halstead would be much quieter, and less disturbed by the air and noise pollution."

Task: Draft a rough route - Group work (~20 mins)

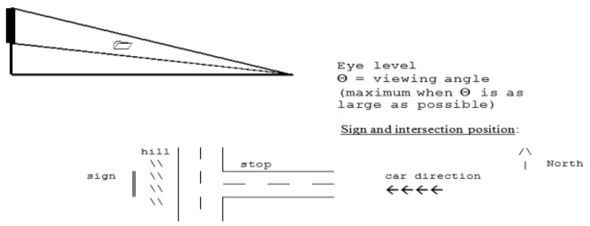
Each group should prioritise the stakeholder issues you consider important. You must come to agreement within your group, by producing a priority list of the interests you consider most important.
Each group should then draft a rough route for a bypass, using the ByPass software.
Keep in mind that you will modify and improve your routes in the next lesson.
It should be clear that in the final lesson you will present your final route and a peer assessment will be taken.
Submit your answers in a file. You'll also share and explain your priority issues and route suggestion in classroom.

Resources: ByPass software, GoogleEarth maps

Generalization: Design bypass for a local town or village that arguably needs one. Then analyse the bypass and estimate its cost and speed limits. You can research the true cost and speed limits of the bypass and find out whether these figures match your own

5. Maximum Viewing Angle

Field of application: Geometry
Required knowledge: Trigonometry, inverse trig. functions, trig. formulas, derivative forms, and extrema theory
Project: Maximum viewing angle
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=5&sesskey=od6JiQPi0G#section-9
Authors:
Coordinator:
The problem: The local municipality will purchase a sign with a local turistic information for advertisement. The



sign will be placed on a hill that is 6 meters high with the bottom of the sign 1,5 meters above the ground on the hill. At what distance from a stopped car at road level should the construction firm place the posts upon which the sign will be fastened so that a maximum viewing angle will be achieved? What will the maximum viewing angle be?

6. Highway survey points

Field of application: Geometry
Required knowledge: Rectangular system of coordinates, angles
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=5
Authors:
Coordinator:
The problem: As a new graduate you have gained employment as a graduate engineer working for a major contractor that employs 2000 staff and has an annual turnover of 600m euros. As part of your initial training period the company has placed you in their engineering surveying department for a six-month period to gain experience of all aspects of engineering surveying. One of your first tasks is to work with a senior engineering surveyor to establish a framework of control survey points for a new 12m euros highway development consisting of a two mile by-pass around a small rural village that, for many years, has been blighted by heavy traffic passing through its narrow main street.
In this exercise you will carry out the geometric calculations that would enable you to determine the precise coordinates of the control survey points. These calculations are based on site measurements obtained through a process known as traversing.
Calculate the coordinates of the traverse points for a section of a control network established prior to the construction of the new highway, data as given in the table below.

Example Data (1):

The coordinates of station A in the local system are defined to be Easting 1000.000 metres and Northing 2000.00 metres. The coordinates of F have been previously calculated by other surveys to be Easting 1558.27 metres and Northing 2253.93 metres. The known initial bearing from A to B is 45° 10' 10''.

Station	**Measured Distance to next station**	**Measured clockwise angle**			**Measured Clockwise angle**	**Easting**	**Northing**
	(metres)	**(degrees**	**mins**	**seconds)**	**(degrees)**	**(metres)**	**(metres)**
A	110.45	45	10	10	* 45.1694	1000.00	2000.00
B	121.33	185	30	30	185.5083		
C	99.86	196	10	24	196.1733		
D	169.27	200	10	25	200.1736		
E	135.26	160	45	45	160.7625		
F							

* This is not an angle observed at A but the known initial bearing from A to B.

Example Data (2):

The coordinates of station A in the local system are defined to be Easting 1306.12 metres and Northing 1888.85 metres. The coordinates of F have been previously calculated by other surveys to be Easting 1397.90metres and Northing 2185.14 metres. The known initial bearing from A to B is 30° 10' 0''.

Station	**Distance to next station**	**Measured clockwise angle**			**Measured clockwise angle**	**Easting**	**Northing**
	(metres)	**(degrees**	**mins**	**seconds)**	**(degrees)**	**(metres)**	**(metres)**
A	98.00	30	10	0	*30.1667	1306.12	1888.85
B	122.35	270	25	35	270.4264		
C	125.46	95	8	15	95.1375		
D	135.67	89	18	22	89.3061		
E	97.36	220	5	55	220.0986		
F							

Resources:. "background" (http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=5&sesskey=od6JiQPi0G#section-11), "questions" (http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=5&sesskey=od6JiQPi0G#section-11)

Practical Activity : Mapping a garden, perimeter of a building or similar area

Preparation: Try to choose an area that is not just a simple rectangle, but if that is not possible, a basketball court could be used, perhaps with some extra objects placed on it whose location has to be marked.
Equipment required: Measuring tape, magnetic compasses, grid paper and geometrical instruments, including ruler and compasses.
Assumed knowledge: Taking bearings using a magnetic compass, mensuration formulae, scale.
Duration: 30 minutes
A. Measure up the area. Draw a rough sketch map of the area. Choose any corner as starting point and label it 1, then number the other corners in sequence, as in the example shown. The number of points will depend on the

shape you are measuring. Use a tape to measure the length of each side. Now use a magnetic compass to find the direction of the longest side (remember that compass bearings are measured clockwise). If all the corners are right angles, you should be able to work out the directions of the other sides, using the fact that all the corners are right angles. If not, use the compass to find the direction of the other sides too.
B. Prepare a table of bearings and distances
Enter the results in the table giving the bearing and distance for each leg (e.g. if there are 10 points – 1 to 2, 2 to 3, … 9 to 10, 10 to 1).
C. Draw a map of the area you have measured
Using your geometry equipment and the data in your table, draw a map of the area on grid paper provided by the surveyor or your teacher. Use a scale of 1 cm to 1 metre, i.e. 1:100. Mark North clearly on your map. Check with a surveyor that you have done this correctly.
D. Use your map to calculate the area of the region.
Show your working and give your answer (in m2 , to 4 significant figures).